Національний технічний університет України
«КИЇВСЬКИЙ ПОЛІТЕХНІЧНИЙ ІНСТИТУТ
імені ІГОРЯ СІКОРСЬКОГО»

Емблема
кафедри
(за наявності)

Department of Computing Technics

HIGHER MATHEMATICS

Syllabus

Requisites of the Course	
Cycle of Higher Education	*First cycle of higher education (Bachelor's degree)*
Field of Study	*12 Information Technologies*
Speciality	*123 Computer Engineering*
Education Program	*Computer Systems and Networks*
Type of Course	*Normative*
Mode of Studies	*full-time*
Year of studies, semester	*1 year (1,2 semester)*
ECTS workload	*2 credits (ECTS). Time allotment - 118 hours 1 semesters, including 54 hours of classroom work, and 20 hours of self-study; 118 hours 1 semesters, including 54 hours of classroom work, and 20 hours of self-study;*
Testing and assessment	*1 semester – Examt, 2 semester – Exam,*
Course Schedule	*3 classes per week by the timetable* http://rozklad.kpi.ua/
Language of Instruction	*English*
Course Instructors	PhD, Senior Lecturer, Mulyk Olena, mobile +380503802967, email mulyk.olena@gmail.com
Access to the course	https://www.youtube.com/playlist?list=PLTHxWFNQpfTJPvNOCaLV98X67kAvmZH1A

Outline of the Course

1. **Course description, goals, objectives, and learning outcomes**

Purpose and tasks of the credit modules "Higher Mathematics-1" and "Higher Mathematics-2":

- logical thinking, development of intellectual abilities;

- education of students of mathematical culture, necessary erudition and intuition in questions of applied application of mathematical knowledge;

- application of mathematical knowledge in the solution of engineering calculations;

- to prove the solution of the problem to a practically acceptable result of the ¬-number, graphics, qualitative conclusion with the use of reference books, tables, computing means;

- independently study literature on mathematics;

- be able to analyze and apply the results obtained.

The main tasks of the credit modules "Higher Mathematics-1" and "Higher Mathematics-2":

- − basic definitions of the functions of one and many variables (area of definition, methods for specifying functions, graphs and properties of elementary functions);
- − boundaries of numerical sequences, boundaries of functions at a point, the first and second important boundaries, the comparison of infinitesimal and properties of equivalent functions, the continuity of functions at the point, the discontinuities of functions;
- − the basis of the differential calculus, the derivative and the differential of the functions of one and many variables, the tangent and normal to the curve, the tangent plane and the normal to the surface, the extremums of functions, the Lopital rule;

abilities:

- − find the boundaries of numerical sequences and functions at the point, compare infinitesimal functions, apply properties equivalent to finding boundaries, investigate the continuity of the function;
- − to differentiate functions, to find differentials of functions of one and many variables, to apply differentials to approximate calculations, to find bounds by the rule of Lopital, to investigate the functions of one variable and to construct schematic diagrams and asymptotes to them, to find extremums of functions of many variables;

experience:

- − learn how to work with information resources, textbooks, reference books, etc .;
- − To master the methods of mathematical analysis for solving problems of corresponding sections of higher mathematics;
- − learn how to solve technical problems obtained as a result of mathematical modeling of processes.

2. Prerequisites and post-requisites of the course (the place of the course in the scheme of studies in accordance with curriculum)

In the structural-logical scheme of the training program in this area, the academic discipline "Higher Mathematics" provides the following disciplines in the program of training specialist: Physics, Engineering and Computer Graphics, Computer Science.

A general course in higher mathematics is the foundation of a specialist in mathematical and engineering education. Mathematical research methods penetrate into all areas of human activity, and because of this, there is a growing interest in the general course of higher mathematics from the side of related sciences, which use a different volume of mathematical knowledge.

3. Content of the course

The main tasks of the cycle of lectures and practical classes: to learn to apply theoretical knowledge to solving practical problems, to develop the skills of working with information resources and to master the methods of mathematical analysis and analytical geometry for solving problems arising as a result of modeling of technical processes.

Definition of a definite integral.
Properties and calculation of a definite integral. **Newton's – Leibniz formula**. Improper integrals. Application of a definite integral.
Integral calculus of functions of many variables.
Double and triple integrals. Definitions, properties, calculations and applications. Replacement of variables in multiple integrals. Jacobian transition. **Curvilinear integrals** of the first and second kind: definition, calculation and application. Surface integrals of the first and second kind: definition, calculation and application.
Ordinary differential equations. Problems leading to the concept of DE. First-order differential equations with separable variables, homogeneous, linear, Bernoulli equations. **Cauchy's problem (IVP)**. Equations that allow lowering the order, **LDE with constant coefficients, special and general right-hand sides.** DE systems.
Numerical Series. Sum of a Numerical Series. Converges and diverges of Series. Necessary Condition for Convergence of a Numerical Series. Sufficient condition for convergence. **Alternating Series.** Leibniz' Theorem. **Power Series.** Interval of Convergence. Integration and Differentiation of Series. Power and Numerical Series for Complex domain.**Taylor's Series and Maclaurin's Series.** Euler's Formula. **Fourier series.** Fourier Series for periodic function $f(x)$ with period 2π. Fourier Series for Even and Odd Functions. The Fourier Series for a Function with Period 2l. On the Expansion of a Nonperiodic Function in a Fourier Series.**Fourier Integral.** The Fourier Integral in Complex Form.

4. Coursebooks and teaching resources

Based:

1. N. Piskunov. Differential and integral calculus.-M.: MIR Publishers., 1969.

   https://pisethsok.files.wordpress.com/2017/11/differential-and-integral-calculus-n-piskunov-editado.pdf

2. Differential calculus of function of one variable.-Repeta Lesya, Mulyk Olena
   https://campus.kpi.ua/tutor/index.php?mode=mob&show&irid=217758

3. Functions of Several Variables \ Arindama Singh\ Department of Mathematics Indian Institute of Technology Madras
   https://campus.kpi.ua/tutor/index.php?mode=mob&show&irid=216191

4. Differential and Integral calculus with examples and applications/George A. Osborne, S.B. – walker professor of mathematics in the Massachusetts instinute of technology. – Boston, USA D.C. Heath & Co Publishers, 1906.

5. Берман Г. Н. Сборник задач по курсу математического анализа / Г. Н. Берман. — С.Пб.: Лань, Специальная литература, 2002. — 448 c. — ISBN 5- 8114-0107-8.

Additions:

1. Understanding Basic Calculus/ S.K. Chung /Department of Mathematics, University of Hong Kong http://math.schung.info/calculus/basic_calculus.html.

2. Диференціальне та інтегральне числення функцій однієї змінної. Практикум. (I курс I семестр) / Уклад.: І. В. Алєксєєва, В. О. Гайдей, О. О. Диховичний, Л. Б. Федорова. — К: НТУУ «КПІ», 2013. — 252 c.

3. Диференціальне та інтегральне числення функцій однієї змінної. Конспект лекцій. (I курс I семестр) / В. О. Гайдей, Л. Б. Федорова, І. В. Алєксєєва, О. О. Диховичний. — К: НТУУ «КПІ», 2013. — 104 с.
4. Письменный Д. Конспект лекций по высшей математике. Полный курс / Д. Письменный. — М.: Айрис-Пресс, 2008. — 608 с. ISBN 978-5-8112-3118-8, 978-5-8112-3480-6.
5. Limits and continuity – A guide for teachers (Years 11–12) Principal author: Peter Brown, University of NSW/ Australian Mathematical Sciences Institute Building 161 The University of Melbourne VIC 3010 Email: enquiries@amsi.org.au Website: www.amsi.org.au
6. Math 221 first semester calculus\2010\The Free Lecture Notes Page (wisc.edu)
7. The Derivative\2010\The Free Lecture Notes Page (wisc.edu)
8. Вища математика: збірник задач: Навч. посібник /В. П. Дубовик, І. І. Юрик, І.П. Вовкодав та ін.; За ред. В. П. Дубовика, І. І. Юрика. – К.: А. С. К., 2005. – 480 с.

Educational content

5. Methodology

The subject of the lecture and a list of the main issues 1 semester:

1	**Lecture 1. Introduction to mathematical analysis.** Basic logical characters. Operation over sets. Valid numbers. Function as a reflection. Inverted display, function graph. **Classroom work 1.** The module of the real number and its properties. Numerical sets. The exact upper and lower bounds of the numerical sets. Newton's Binom Formula. **Self-study:** [5, No. 178, 246-256 paired.]
2	**Lecture 2. The limit of the numerical sequence.** Numerical sequences. Definitions of the limit of numerical sequence. Important theorems on the sequence limit. [2, pages 4-10.] **Classroom work 2.** Properties of sequences and properties of limits.
3	**Lecture 3. Infinitely small and infinitely large sequences.** Signs of limit existence. Properties of numerical sequences. Number *e.* Weierstrass theorem on the existence of a boundary of a monotone bounded sequence. [1, pages 52-55], [2, pages 10-15.] **Classroom work 3.** Uncertainties and their types. **Self-study:** [5, No. 260-266.] **Problems 1-4**. Natural logarithms.[1, 56]
4	**Lecture 4. The limit of the function.** Definition of the limit of the function in Heine and Cauchy. Conditions of the existence of the limit of the function. Theorems on the limit of the function (on the preservation of the sign, arithmetic, transition to the limit in inequalities). Infinitely small and infinitely large functions. [2, pages 17-22.] **Classroom work 4. The limit of the function**. Limit at the points: Substitution Method, Conjugate Method, Limits at infinity. **Problem 5-6.** **Self-study:** [5, No. 268-308 are paired]. Unilateral limits of a function, the definition of an infinite limit of a function. Hyperbolic functions.

5	**Lecture 5. First and second important limits.** [, pages 23-26.]

Classroom work 5. First and second important limits. [2, pages 23-26.], **Problems 7-8.**

Self-study: [5, No. 351-376 (even numbers)]. Recommended literature [1, Ch.4. §4.]

6	**Lecture 6.** Comparison of infinitely small functions. Comparison of infinitesimal functions, Applying equivalences to calculating limits of functions.

Classroom work 6. Comparison of infinitesimal functions. Substitution method. Applying equivalences to calculating limit functions.

Self-study: [5, No. 402-414], [2, pages 27-28], **Problems 9**.

Recommended literature [1, pages 63-66]

7	**Lecture 7. Continuity of the function in the point.** Theorems on continuous functions in a point. Types of discontinuity. Breakpoints, their types. Classification of breakpoints. **Continuity of the function on the segment.** Theorems on continuous functions on a segment: Weierstrass, Cauchy, Bolzano-Cauchy. Theorem on the continuity of an inverse function. [1, pages 57-61]

Classroom work 7. Continuity of the function. Breakpoints, their types. Properties of continuous functions. [, pages 29-34], **Problems 10.**

Self-study: [5, No. 402-414, 222-240 (even numbers)]. Proof of the Weierstrass theorems, Cauchy theorems.

Recommended literature [1, pages 57-63]

8	**Lecture 8. The derivative.** The definition of the derivative. Definition of the derivative at the point. Unilateral Derivatives. Continuity and existence of the derivative at the point. Geometric, physical content of the derivative. The tangent and normal to the function graph. [1, pages 70-80, 126-129]

Classroom work 8. Differentiability of functions. The tangent and normal to the function graph. Basic rules of differentiation. [2, pages 39-47], **Problem 11**.

Self-study: [5, No. 517-632, (even numbers)]. Table of derivatives. [1, pages 102-103]

9	**Lecture 9. Differentiation of functions.** Derivative of inverse, composite functions, composite exponential function, functions given implicitly and parametrically. Derivative of power-index function. [1, pages 84-110].

Classroom work 9. Differentiation of functions. Derivative of inverse, composite functions, functions given implicitly and parametrically. [2, pages 47-57], **Problem 12, 13,** [5, No. 776-786, 792-812, 932-940 (even numbers)].

Self-study: Basic rules of differentiation. The derivative of a logarithmic function. Derivatives of hyperbolic functions [1, pages 110-113], an inverse function and its differentiation [1, pages 93-97].

Recommended literature [1, pages 84-113].

10   **Lecture 10. Derivatives of higher orders.** Derivatives of higher orders of functions given explicitly, implicitly, and parametrically. Formula Leibniz. [1, pages 119-126]

Classroom work 10. Derivatives of higher orders. Derivatives of higher orders of functions given explicitly, implicitly, and parametrically. [2, pages 62-68]

Self-study: [5, No. 1026, 1029, 1031, 1039, 1058-1062, 1070-1078 (even numbers)], Formula of approximate calculations.

11   **Lecture 11. Differential of the functions.** The meaning of differential and geometric meaning. Properties of the differential, invariance of the form of the first differential. [1, pages 113-118]

Classroom work 11. Differential of the functions. The meaning of differential and applications. Properties of the differential. [2, pages 59-61]

12   **Lecture 12. Differentiability of the function.** Differentials of higher orders. Non-invariance of the form of the second differential. **Basic theorems of differential calculus.** Theorems of Lagrange, Role, Cauchy. [1, pages 141-145], [2, pages 69-72]

Classroom work 12. Differentiation of functions. Higher-order derivative of inverse, composite functions, functions given implicitly and parametrically. Derivative of power-index function. [2, pages 62-68], **Problems 14-15**.

Self-study: [5, No. 776-786, 792-812, 932-940 even]. Differentials of higher orders of implicitly given functions.

Recommended literature [3,4]

13   **Lecture 13. Basic theorems of differential calculus.** Applications of theorems of Lagrange, Role, Cauchy. Rule of Lopital. Application of Lopital`s rule. [1, pages 145-153]

Classroom work 13. Rule of Lopital. Application of Lopital`s rule. [2, pages 72-76], [5, No. 1129, 1136, 1199, 1324-1334, 1356-1362 (even numbers)], **Problem 16**.

14   **Lecture 14. Taylor's formula.** Taylor's formula. Remainder of the Taylor formula in the form of Lagrange, Peano. Applying the Taylor formula to calculating approximate values and boundary functions. [1, pages 153-160]

Classroom work 14. Applycation of the Taylor and MacLoren formula. MacLoren formula of basic elementary functions. [2, pages 77-82], [5, No. 1498, 1503, 1504, 1507, 1511, 1522, 1527, 1528], **Problem 17**.

Self-study: Proof of Taylor's formula. [1, page 155]

27   **Lecture 27: Integrals of irrational functions.** Integration of certain irrational functions by means of trigonometric substitutions [1, pages 372-378, 384-386]

      Classroom work 27. Integrals of irrational functions. [1, pages 372 -378, No. 112-124 (even numbers), page 391, No. 148-151, page 392, No. 170-172, page 392, No. 196-216 (even numbers), page 394]

      Self-study: Integration of binomial differentials. [1, pages 376-378] Functions whose integrals cannot be expressed in terms of elementary functions. [1, pages 386-387, No. 189-191, page 394]

The subject of the lecture and a list of the main issues 2 semester:

1   **Lecture 1: The definite integral.** Statement of the problem. The lower and upper integral sums. Basic properties of the definite integral. [1, pages 396-407]
      Classroom work 1: The definite integral. Evaluation of integral. [1, pages 402-403], [5, No. 1628, 1636, 1637, 1658, 1663]

2   **Lecture 2: Newton-Leibniz formula**. Changing the variable in the definite integral. Integration by parts. [1, pages 407-416]
      Classroom work 2: Changing the variable in the definite integral. Integration by parts. [5, No. 1672, 1673, 1637, 1658, 1663]

3   **Lecture 3:** Improper Integrals. [1, pages 416-424]
      Classroom work 3: Improper Integrals I and II kind. [5, No.2426-2435 (even numbers)]
      Self-study 3: Integrals Dependent on a Parameter. [1, pages 435-438]

4   **Lecture 4: Geometric application of the definite integral.** The Area in Rectangular Coordinates and the Area of a Curvilinear Sector in Polar Coordinates.
      [1, pages 442--447]
      Classroom work 4: Polar Coordinates. Computing Areas in Rectangular Coordinates. The Area of a Curvilinear Sector in Polar Coordinates.[5, No. 2456-2466 (even numbers), 2490-2493, 2496-2500]
      Self-study 4: Approximating Definite Integrals. Chebyshev's Formula.
      [1, pages 424--435]

5   **Lecture 5: Geometric and mechanical application of the definite integral.** The Arc Length of a Curve. The Volume of a Solid of Revolution. The Surface of a Solid of Revolution. Coordinates of the Centre of Gravity [1, pages 447—462]
      Classroom work 5: The Arc Length of a Curve. The Volume of a Solid of Revolution. The Surface of a Solid of Revolution. [5, No. 2520-2523, 2531-2533, 2536, 2556-2558]
      Self-study 5: Computing the Volume of a Solid from the Areas of Parallel Sections (Volumes by Slicing) [1, pages 453-455]

6   **Lecture 6: Multiple integrals.** Double Integrals. Calculating Double Integrals.
      [1, pages 608-623]
      Classroom work 6: Calculating Double Integrals. [1, pages 608-623], [5, No. 3478-3484 (even numbers), 3298-3504 (even numbers), 3508-3512]

7	**Lecture 7: Application of the Double Integrals.** Calculating Areas and Volumes by Means of Double Integrals. Changing Variables in a Double Integral. The Double Integral in Polar Coordinates.
	Classroom work 7: Calculating Areas and Volumes by Means of Double Integrals. The Double Integral in Polar Coordinates. [5, No.3526-3528, 3532,3533, 3539, 3563-3565]
	Self-study 7: [5, No. 3566-3568]
8	**Lecture 8: Triple Integrals.** Evaluating a Triple Integral. Change of Variables in a Triple Integral. [1, pages 650-660]
	Classroom work 8: Triple integral in cylindrical coordinates. A triple integral in spherical coordinates. [1, pages 656-659], [5, No. 3548-3554, 3610-3618 (even numbers)]
	Self-study: General change of variables in the triple integral. [1, pages 659-660]
9	**Lecture 9: Geometric Application of Double and Triple Integrals.** The Moment of Inertia of the Area of a Plane Figure. The Coordinates of the Centre of Gravity of the Area of a Plane Figure. The Moment of Inertia and the Coordinates of the Centre of Gravity of a Solid. [1, pages 643-650, 660-662]
	Classroom work 9: The Moment of Inertia of the Area of a Plane Figure. The Coordinates of the Centre of Gravity of the Area of a Plane Figure. The Moment of Inertia and the Coordinates of the Centre of Gravity of a Solid. [5, No.3648, 3653, 3668]
	Self-study: Computing the Area of a Surface. [1, pages 638-642]
10	**Lecture 10: Line integral of the I kind.** Evaluating a Line Integral of the I kind [1, pages 447-653]
	Classroom work 10: Application of Line Integral of the I kind. [5, No.3770-3774, 3782, 3783, 3784, 3786, 3789]
11	**Lecture 11: Line integral of the II kind.** Evaluating a Line Integral of the I kind [1, pages 670-679]
	Classroom work 11: Application of Line Integral of the II kind. [5, No.3806-3816 (even numbers)]
	Self-study: Connection between Line Integral I and II kind.
12	**Lecture 12: Green's Formula.** Conditions for a Line Integral Being Independent of the Path of Integration. [1, pages 679-687]
	Classroom work 12: Application of Line Integral of the II kind. Computing the work of a variable force F on some curved path L. [5, No. 3822-3825, 3838-3840, 3843, 3845, 3846, 3862]
	Self-study: [5, No. 3855, 3856]
13	**Lecture 13: Differential equations.** Statement of the Problem. The Equation of Motion of a Body with Resistance of the Medium Proportional to the Velocity. First-Order Differential Equations. Equations with Separated and Separable Variables. The Problem of the Disintegration of Radium. [1, pages 469-478]
	Classroom work 13: First-Order Differential Equations. Equations with Separated and Separable Variables. [5, No. 3907 – 3910, 3913-3916]
	Self-study: [5, No. 3917, 3919]

21	**Lecture 21: Alternating Series.** Leibniz' Theorem. Pfus-and-Minus Series. Absolute and Contitional Convergence. **Functional Series.** Majorised Series. [1, pages 727-739] **Classroom work 21:** Alternating Series. Application of Leibniz' Theorem. Pfus-and-Minus Series. [5, No. 2790-2798 (even numbers)] **Self-study:** [5, No.2835], The Continuity of the Sum of a Series [1, pages 736-739]
22	**Lecture 22: Power Series.** Interval of Convergence. Integration and Differentiation of Series. Differentiation of Power Series. [1, pages 739-750] **Classroom work 22:** Interval of Convergence. Differentiation of Power Series. [5, No. 2802-2816 (even numbers)] **Self-study:** Series in Powers of (x- a) [1, pages 748-750]
23	**Lecture 23: Taylor's Series and Maclaurin's Series.** Euler's Formula. The Binomial Series. Integrating Differential Equations by Means of Series. **Classroom work 23:** Examples of Expansion of Functions in Series. Expansion of the Function $\ln (1 + x)$ in a Power Series. Integration by use of Series (Calculating Definite Integrals). [5, No. 2827-2830, 2842-2868 (even numbers)], [1, No. 39-42 on page 769, 64-67 on page 771, 79-81 on page 772, 105-107 on page 773] **Self-study:** Computing Logarithms. [1, pages 756-758], [1, No. 84-94 on page 772 (even numbers)]
24	**Lecture : Fourier series.** Statement of the Problem. Expansions of Functions in Fourier Series. A Remark on the Expansion of a Periodic Function in a Fourier Series [1, pages 776-787] **Classroom work :** Fourier Series for periodic function $f(x)$ with period 2π. [1, No. 4-7 on page 813]
25	**Lecture 25: Fourier series.** Fourier Series for Even and Odd Functions. The Fourier Series for a Function with Period 2l. On the Expansion of a Nonperiodic Function in a Fourier Series. [1, pages 787-798] **Classroom work 25:** Fourier Series for Even and Odd Functions. The Fourier Series for a Function with Period 2l. Nonperiodic Function in a Fourier Series. [1, No. 8-13 on page 813-814] **Self-study:** [1, No. 14 on page 814]
26	**Lecture 26: Fourier Integral.** The Fourier Integral in Complex Form. [1, pages 810-812] **Classroom work 26:** Application of the Fourier Integral.
27	**Lecture 27: Final lesson. Course review.** **Classroom work 27: Exam preparation.**

6. Self-study

Students have to do homework and solve Google tests in class, read the specified additional literature. By the end of each semester control they must pass (send by email) homework in the number of tasks specified by the teacher.

7. Course policy

At the beginning of teaching the lecture material on a new theme, it is desirable to give a complete and complete description of the section and topics, to provide the keywords and basic concepts that will be considered. Further, detail the material, provide strict definitions, formulate theorems on this topic and, if possible, prove it. Invite students to prove some facts on their own. To illustrate the theoretical material with examples. Pay particular attention to the key points of the decision.

It is appropriate to offer students the opportunity to independently examine some issues of the topic of lectures, indicate textbooks and information resources, where it is possible to get acquainted with the concepts introduced, to present historical facts that led to the emergence of new concepts.
Each practical lesson is conducted only after considering the relevant topic at the lecture. According to the joint desire of students and lecturers, it is possible to hold a problem lecture or a lecture in the form of scientific dissent.

For participation in the faculty olympiad of higher mathematics, 1 point is added, prizes - 2 points, the winner - 3 points.

8. Monitoring and grading policy

At the first class the students are acquainted with the grading policy which is based on Regulations on the system of assessment of learning outcomes https://document.kpi.ua/files/2020_1-273.pdf

Rating system of evaluation:

Student rating from the credit module "Higher mathematics-1. **"Differential calculus of function of a one real variable"** in the discipline "HIGHER MATHEMATICS" consists of the points that he/she receives for:

1) current work in practical classes - 16 points - 8 responses (each student on average) for 24 practical classes (each student must complete each Google test by more than 60%) (R1);

2) 3 modular control works - 24 points (R2):

a) The limit of the function (8 points).

b) The derivative of a function of one variable (8 points).

c) Functions of many variables and indefinite integrals (8 points).

3) performing a typical calculation work - 10 points (R3);

4) written exam paper - 50 points (R4):

The student's rating in the course consists of points that he/she receives for participation in 24

$$Rs=R1+R2+R3=100 \text{ points}$$

Student rating from the credit module "Higher mathematics-2 "**Calculus of function of several variables. Differential equations. Series**" in the discipline "HIGHER MATHEMATICS" consists of the points that he/she receives for:

1) current work in practical classes - 16 points - 8 responses (each student on average) for 24 practical classes (each student must complete a Google test by more than 60%) (R1);

2) 3 modular control works - 24 points (R2):

a) The definite integral, Multiple integrals, Line integrals (10 points).

b) Differential equations (7 points).

c) Series (7 points).

3) performing a typical calculation work - 10 points (R3);

4) written exam paper - 50 points (R4).

The student's rating in the course consists of points that he/she receives for participation in 24

<h3 style="text-align:center">Rs=R1+R2+R3=100 points</h3>

Written exam paper:

At the exam, the student conducts a written examination work. Each ticket consists of 1 theoretical question and 4 practical tasks. The list of theoretical questions is given by the examiner at the last discipline.

Weighted score -10. Maximum score 10 points x 5 tasks = 50 points.

- The theoretical question - 10 points;

- Task 1 with topic Introduction to the analysis - 10 points;
- Task 2 of the topic Differential numbering of functions of one variable - 10 points;
- Task 3 of the topic Investigating the functions of one variable - 10 points;
- Task 4 of the topic Differential number of functions of many variables - 10 points.

The system of evaluation of theoretical questions:

– "Excellent", full answer (not less than 90% of the required information) - 9-10 points;

– "good", sufficiently complete answer (not less than 75% of the required information), or minor inaccuracies - 7-8 points;

− "satisfactory", incomplete answer (not less than 60% of the required information) and some errors - 6 points;

− "unsatisfactory", unsatisfactory answer - 0-5 points.

Practical task evaluation system:

− "excellent", complete unblemished solution of the task - 10 points;

− "good", complete solving of the task with minor inaccuracies - 8-9 points;

− "satisfactory", the task is fulfilled with certain disadvantages - 6-7 points;

− "unsatisfactory", unsatisfactory response, wrong method of resolution - 0-5 points.

According to the university regulations on the monitoring of students' academic progress (https://kpi.ua/document_control) there are two assessment weeks, usually during 7th/8th and 14th/15th week of the semester, when students take the Progress and Module tests respectively, to check their progress against the criteria of the course assessment policy.

The students who finally score the required number of points (≥60) can:
- get their final grade according to the rating score;
- perform a Fail/Pass Exam in order to increase the grade.

Students whose final performance score is below 60 points but more than 30 are required to complete a Fail/Pass Exam. If the grade for the test is lower than the grade, which the student gets for his semester activity, a strict requirement is applied - the student's previous rating is canceled and he receives a grade based on the results of the Fail/Pass Exam. Students whose score is below 30 are not allowed to take the Fail/ Pass Exam.

The final performance score or the results of the Fail/ Pass Exam are adopted by university grading system as follows:

Score	Grade
100-95	Excellent
94-85	Very good
84-75	Good
74-65	Satisfactory
64-60	Sufficient
Below 60	Fail
Course requirements are not met	Not Graded

9. Additional information about the course
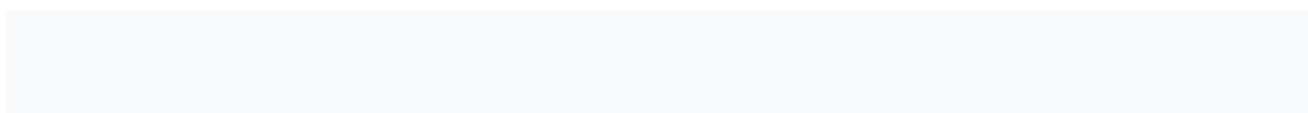
Exam problems I semester (theoretical questions):

1. **The limit of numerical sequence**.
   - Definitions of the limit of numerical sequence.
   - Limit of the numerical sequence: $\lim_{n\to\infty} X_n = A$.
   - Important theorems on the sequence limit theorem (*existence criterion of limit of numerical sequence*).
   - Properties of sequences and properties of limits.
   - Euler number e.
2. **The limit of the function.**
   - The formal definition of the limit.
   - Properties of the limit of function.
   - Calculation of function limits.
3. **Basic theorems of limits of the function.**
   - The first important limit.
   - The second important limit.
   - Comparison of infinitesimal functions.
4. **Continuous functions.**
   - Definition of continuous functions at the point.
   - Types of discontinuities.
   - Properties of continuous functions.
5. **The derivative.**
   - The definition of the derivative.
   - The existence of a derivative.
   - Geometric meaning of the derivative.
   - Finding derivative functions.
   - Implicit differentiation.
   - Logarithmic differentiation.
   - Parametric differentiation.
6. **The differential.**
   - The first order differential.
   - Differentials of higer orders.
7. **Higher order derivatives.**
   - Higher order derivatives of explisit functions.
   - Leibnitz's formula.
   - Higher order parametric differentiation.
8. **Basic theorems of the differential calculus.**
   - Intermediate value theorem and The extreme value theorem.
   - Rolle's theorem.
   - Lagrange`s theorem (mean value theorem).
   - Cauchy theorem.
   - L'hopital's theorem (L'hôpital's rule).
9. **The taylor and the maclaurin formulas.**
   - The Taylor` s polynomial and remainder term**.**
   - The Taylor` s polynomial and remainder term.
   - Table of important maclaurin formula for reference.
10. **Investigation of functions using derivatives.**
    - Increasing and decreasing functions.

- Extrema and the first derivative test.
- Absolute extrema.

11. The second derivative for investigation of functions.
- The second derivative test for a local maximum or minimum.
- The second-derivative test for concavity.
- Inflection points.

12. Asymptote.

13. Graphing a function

14. Complex numbers.
- Basic operations on complex numbers.
- Euler's formula. De Moivre's formula.

15. Functions of many variables.
- Definition, definition area. The limit and continuity of functions of many variables.
- Differentiation of functions of many variables.
- Derivatives of higher orders of functions of many variables.
- Full differentials of higher orders.

16. Directional derivative.
- Tangential plane and normal to the surface.
- Gradient of the scalar field.

17. Extreme functions of many variables.

18. Conditional extremum.

19. Antiderivative and the indefinite integral.
- Integration by substitution.
- Table of integrals.

20. Integration.
- Integrals of functions containing a quadratic trinomial.
- Integration by parts.
- Partial rational fractions and their integration.

21. Integrals of irrational functions.
- Integrals of irrational functions.
- Integration of certain irrational functions by means of trigonometric substitutions.
- Integration of binomial differentials.

Exam problems II semester (theoretical questions):

1. The definite integral.
- Statement of the problem.
- The lower and upper integral sums.
- Basic properties of the definite integral.

2. Newton-Leibniz formula.
- Changing the variable in the definite integral.
- Integration by parts.
- Changing the variable in the definite integral.

3. Improper Integrals.
- Improper Integrals I kind.
- Improper Integrals I kind.

4. **Geometric application of the definite integral.**

- The Area in Rectangular Coordinates.
- Area of a Curvilinear Sector in Polar Coordinates.

5. Geometric and mechanical application of the definite integral.
- The Arc Length of a Curve.
- The Volume of a Solid of Revolution.
- The Surface of a Solid of Revolution.
- Coordinates of the Centre of Gravity.

6. Double Integrals.
- Basic properties of the double integral.
- Calculating Double Integrals.

7. Application of the Double Integrals.
- Calculating Areas and Volumes by Means of Double Integrals.
- Changing Variables in a Double Integral.
- The Double Integral in Polar Coordinates.

8. Triple Integrals.
- Basic properties of the triple integral.
- Evaluating a Triple Integral.
- Change of Variables in a Triple Integral.

9. Triple Integrals.
- Triple integral in cylindrical coordinates.
- Triple integral in spherical coordinates.

10. Geometric Application of Double and Triple Integrals.
- The Moment of Inertia of the Area of a Plane Figure.
- The Coordinates of the Centre of Gravity of the Area of a Plane Figure.
- The Moment of Inertia and the Coordinates of the Centre of Gravity of a Solid.

11. Line integral of the I kind.
- Basic properties of the Line Integral of the I kind.
- Evaluating a Line Integral of the I kind.
- Application of Line Integral of the I kind.

12. Line integral of the I I kind.
- Basic properties of the Line Integral of the II kind.
- Evaluating a Line Integral of the II kind.
- Application of Line Integral of the II kind.

13. Green's Formula.
- Conditions for a Line Integral Being Independent of the Path of Integration.
- Computing the work of a variable force F on some curved path L.

14. Differential equations.
- Statement of the Problem.
- First-Order Differential Equations.
- The initial value problem.
- Equations with Separated and Separable Variables.

15. Homogeneous First-Order Equations.
- Equations Reducible to Homogeneous Equations.
- The initial value problem.
- Solution of linear equation.

16. First-Order Differential equations.
- First-Order Linear Differential equations.
- Bernoulli's Equation.
- Exact Differential Equations.

17. Higher-order differential equations.
- An Equation of the Form $y^{(n)} = f(x)$.
- Some Types of Second-Order Differential Equations Reducible to First-Order Equations.
- Higher-order differential equations which allow lowering the order.

18. Homogeneous Linear Equations.
- Definitions and General Properties.
- The initial value problem.
- Second-Order Homogeneous Linear Equations with Constant Coefficients.
- Homogeneous Linear Equations of the n-th Order with Constant Coefficients.

19. Nonhomogeneous Linear Equations with Constant Coefficients.
- Nonhomogeneous Second-Order Linear Equations with Constant Coefficients.
- Nonhomogeneous Linear Equations of the n-th Order with Constant Coefficients.
- The method of variation of arbitrary constants.
- The initial value problem.

20. Nonhomogeneous Linear Equations with Constant Coefficients.
- The method of variation of arbitrary constants for nonhomogeneous Second-Order Linear Equations with Constant Coefficients.
- The initial value problem.

21. Systems of Ordinary Differential Equations.
- Systems of Linear Differential Equations with Constant Coefficients.

22. Numerical Series.
- Sum of a Numerical Series.
- Partial sum of Series.
- Converges and diverges of Series.
- Necessary Condition for Convergence of a Numerical Series.

23. Sufficient condition for convergence.
- Comparing Series with Positive Terms.
- D'Alembert's Test.
- Cauchy's Test.
- The Integral Test for Convergence of a Series.

24. Alternating Series.
- Leibniz' Theorem.
- Pfus-and-Minus Series.
- Absolute and Contitional Convergence.
- Application of Leibniz' Theorem.

- Numerical Series for Complex domain.

25. Power Series.
- Interval of Convergence.
- Integration and Differentiation of Series.
- Differentiation of Power Series.
- Power Series for Complex domain.

26. Taylor's Series and Maclaurin's Series.
- Euler's Formula.
- The Binomial Series.
- Integrating Differential Equations by Means of Series.
- Expansion of the Function $\ln (1 + x)$ in a Power Series.
- Integration by use of Series (Calculating Definite Integrals).

27. Fourier series.
- Statement of the Problem. Properties of the Fourier series.
- Expansions of Functions in Fourier Series.
- Fourier Series for periodic function $f(x)$ with period 2π.

28. Fourier series.
- Fourier Series for Even and Odd Functions.
- The Fourier Series for a Function with Period 2l.
- On the Expansion of a Nonperiodic Function in a Fourier Series.

29. Fourier Integral.
- Properties of the Fourier Integral.
- The Fourier Integral in Complex Form.

Syllabus of the course

Is designed by teacher PhD, Senior Lecturer Mulyk Olena.

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Approved by the Faculty Board of Methodology (protocol № 7 , 20 June 2021)